



ELSEVIER

Journal of Magnetism and Magnetic Materials 201 (1999) 144–148

M Journal of
M magnetism
M and
magnetic
materials

www.elsevier.com/locate/jmmm

Ferrofluid flows in AC and traveling wave magnetic fields with effective positive, zero or negative dynamic viscosity

Markus Zahn*, Loretta L. Pioch

*Massachusetts Institute of Technology, Department of Electrical Engineering and Computer Science,
Laboratory for Electromagnetic and Electronic Systems, Cambridge, MA 02139, USA*

Received 1 June 1998; received in revised form 7 August 1998

Abstract

Analysis and measurements have shown anomalous behavior of ferrofluids in AC magnetic fields, whereby in a rotating magnetic field the ferrofluid can be pumped but the flow direction can reverse as a function of magnetic field amplitude, frequency, and direction. This anomalous behavior is investigated using the governing fluid mechanical linear and angular momentum conservation equations including non-symmetric viscous and Maxwell stress tensors. Here we examine a simple case where applied magnetic field components transverse and parallel to the duct axis are spatially uniform and vary sinusoidally with time. The governing equations are numerically integrated to solve for flow and spin velocity distributions. Numerical integrations show the highly non-linear and multi-valued solutions for flow and spin velocities. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Ferrofluids; Magnetic field; Dynamic viscosity

1. Introduction

The motion of ferrofluid in a traveling wave magnetic field has been paradoxical as many investigators find critical magnetic field strength and frequency ranges where the fluid moves opposite to the direction of the traveling wave (backward pumping) while outside these ranges the ferrofluid moves in the same direction (forward pumping) [1–6]. Under AC magnetic fields, fluid viscosity acting on the magnetic particles suspended in the ferrofluid causes the magnetization \mathbf{M} to lag behind a traveling \mathbf{H} . With \mathbf{M} not collinear with \mathbf{H} , there is a body torque density $\mathbf{T} = \mu_0(\mathbf{M} \times \mathbf{H})$ acting on the ferrofluid even in a uniform magnetic field when the magnetization force density along the duct axis is zero.

* Corresponding author. Fax: + 1-617-258-6774.

E-mail address: zahn@mit.edu (M. Zahn)

2. Magnetic equations

The magnetization relaxation equation with ferrofluid undergoing simultaneous magnetization and reorientation due to fluid convection at velocity \mathbf{v} and particle spin at angular velocity $\boldsymbol{\omega}$ is [1,3–7]

$$\frac{\partial \mathbf{M}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{M} - \boldsymbol{\omega} \times \mathbf{M} + \frac{1}{\tau} [\mathbf{M} - \chi_0 \mathbf{H}] = 0, \quad (1)$$

where τ is a relaxation time constant and χ_0 is the effective magnetic susceptibility which in general can be magnetic field dependent but in this paper will be taken to be constant.

We apply Eq. (1) to a planar ferrofluid layer confined between rigid walls of width d . The imposed axial magnetic field H_z and transverse magnetic flux density B_x are spatially uniform and are imposed on the system by external sources. Because the imposed fields are uniform with the y and z coordinates, field components can only vary with the x coordinate. Gauss’s law for the magnetic flux density and Ampere’s law for the magnetic field intensity with zero current density then require the imposed fields to be uniform throughout the ferrofluid, independent of x . The total magnetic field \mathbf{H} and magnetic flux density \mathbf{B} inside the ferrofluid layer vary sinusoidally in time and are of the form

$$\mathbf{B} = \Re\{[\hat{B}_x \mathbf{i}_x + \hat{B}_z(x) \mathbf{i}_z] e^{j\Omega t}\}, \quad \mathbf{H} = \Re\{[\hat{H}_x(x) \mathbf{i}_x + \hat{H}_z \mathbf{i}_z] e^{j\Omega t}\}, \quad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \quad (2)$$

The magnetic force and torque densities are

$$\mathbf{f} = \mu_0(\mathbf{M} \cdot \nabla) \mathbf{H}, \quad \mathbf{T} = \mu_0(\mathbf{M} \times \mathbf{H}) = \mu_0(-M_x H_z + M_z H_x) \mathbf{i}_y. \quad (3)$$

Along the duct (z) axis, H_z is uniform so that $f_z = 0$. In the small spin velocity limit where $(\omega_y \tau \ll 1)$, the approximate time-average torque density is

$$\lim_{\omega_y \tau \ll 1} \langle T_y \rangle \approx T_0 + \alpha \omega_y, \quad (4)$$

where

$$T_0 = - \frac{\chi_0 \Re\{[\chi_0(\Omega\tau)^2 + j\Omega\tau((\Omega\tau)^2 + 1 + \chi_0)][\hat{H}_z \hat{B}_x^*]\}}{[1 + \chi_0 + (\Omega\tau)^2]^2 + (\chi_0 \Omega\tau)^2},$$

$$\alpha = \frac{(\chi_0 \tau / 2)[(|\hat{B}_x|^2 / \mu_0)[(\Omega\tau)^2 - 1] + \mu_0 |\hat{H}_z|^2 [(\Omega\tau)^2 - (1 + \chi_0)^2]}{[1 + \chi_0 + (\Omega\tau)^2]^2 + (\chi_0 \Omega\tau)^2}. \quad (5)$$

Note that $T_0 = 0$ if either H_z or B_x is zero.

3. Fluid mechanical equations

In the negligible inertia, viscous dominated limit the flow and spin momentum equations with zero spin viscosity reduce to

$$(\zeta + \eta) \frac{d^2 v_z}{dx^2} + 2\zeta \frac{d\omega_y}{dx} - \frac{\partial p}{\partial z} = 0, \quad -2\zeta \left(\frac{dv_z}{dx} + 2\omega_y \right) + \langle T_y \rangle = 0. \quad (6)$$

In the small spin velocity limit of Eq. (4) ($\omega_y \tau \ll 1$) the approximate flow and spin velocity profiles are

$$v_z(x) \approx \frac{x(x-d)}{2\eta_{\text{eff}}} \frac{\partial p}{\partial z}, \quad \omega_y(x) \approx \frac{1}{(4\zeta - \alpha)} \left[T_0 - \frac{\zeta(2x-d)}{\eta_{\text{eff}}} \frac{\partial p}{\partial z} \right], \quad \eta_{\text{eff}} = \eta - \frac{\alpha\zeta}{4\zeta - \alpha}, \quad (7)$$

where η_{eff} is the effective viscosity [4–7]. Depending on the value of α , η_{eff} can be positive, zero, or negative. When $\eta_{\text{eff}} = 0$, ω_y becomes infinite, violating the small spin velocity approximation.

4. Representative solutions for spin and linear velocities

Magnetic field amplitude, frequency, and direction cause the ferrofluid effective viscosity of Eq. (7) to be different than the dynamic viscosity η of the carrier fluid. Here we consider two special cases of linearly polarized magnetic fields either purely transverse to the duct axis so that $|\hat{B}_x| = \mu_0 H_0$, $|\hat{H}_z| = 0$, or purely parallel to the duct axis, $|\hat{B}_x| = 0$, $|\hat{H}_z| = H_0$. To further simplify the algebraic analysis we consider the special case when the vortex and dynamic viscosities are equal, $\eta = \zeta$. Then solving Eq. (7) for the parameter α of Eq. (5) in terms of $\zeta = \eta$ and η_{eff} yields

$$\alpha = \frac{4\zeta(\zeta - \eta_{\text{eff}})}{2\zeta - \eta_{\text{eff}}} = \frac{\chi_0 \tau}{2} \frac{[(\Omega\tau)^2 - 1]\mu_0 H_0^2}{[(1 + \chi_0 + (\Omega\tau)^2)^2 + (\chi_0 \Omega\tau)^2]}, \quad |\hat{B}_x| = \mu_0 H_0, \quad |\hat{H}_z| = 0,$$

$$\alpha = \frac{4\zeta(\zeta - \eta_{\text{eff}})}{2\zeta - \eta_{\text{eff}}} = \frac{\chi_0 \tau}{2} \frac{[(\Omega\tau)^2 - (1 + \chi_0)^2]\mu_0 H_0^2}{[(1 + \chi_0 + (\Omega\tau)^2)^2 + (\chi_0 \Omega\tau)^2]}, \quad |\hat{B}_x| = 0, \quad |\hat{H}_z| = H_0. \quad (8)$$

Then solving Eq. (8) for $\Omega\tau$ results in a fourth-order biquadratic equation

$$(\Omega\tau)^4 + b(\Omega\tau)^2 + c = 0 \quad (9)$$

with solutions

$$(\Omega\tau)^2 = \frac{-b \pm \sqrt{b^2 - 4c}}{2}, \quad b = \left[(1 + \chi_0)^2 + 1 - \frac{\chi_0 \mu_0 H_0^2 \tau}{2\alpha} \right], \quad c = \left[(1 + \chi_0)^2 + \frac{\chi_0 \mu_0 H_0^2 \tau}{2\alpha} \right],$$

$$|\hat{B}_x| = \mu_0 H_0, \quad |\hat{H}_z| = 0,$$

$$(\Omega\tau)^2 = \frac{-b \pm \sqrt{b^2 - 4c}}{2}, \quad b = \left[(1 + \chi_0)^2 + 1 - \frac{\chi_0 \mu_0 H_0^2 \tau}{2\alpha} \right], \quad c = \left[(1 + \chi_0)^2 \left(1 + \frac{\chi_0 \mu_0 H_0^2 \tau}{2\alpha} \right) \right],$$

$$|\hat{B}_x| = 0, \quad |\hat{H}_z| = H_0. \quad (10)$$

The non-dimensional frequency $\tilde{\Omega}$ versus $\tilde{\zeta}$ for various values of positive, zero and negative $\tilde{\eta}_{\text{eff}}$ is plotted in Fig. 1 for the case of a purely axial magnetic field so that $T_0 = 0$. Solutions for negative $\tilde{\eta}_{\text{eff}}$ fall in a region between the positive viscosity solutions. Note that for a given $\tilde{\Omega}$ and positive $\tilde{\eta}_{\text{eff}}$ that there are two possible values of $\tilde{\zeta}$ while for a negative $\tilde{\eta}_{\text{eff}}$ there is only one possible $\tilde{\zeta}$.

Once values of $\tilde{\zeta}$ and $\tilde{\eta}_{\text{eff}}$ are chosen, the value of frequency $\tilde{\Omega}$ is determined from Fig. 1. To determine the spin velocity spatial distribution by numerically integrating Eq. (6), it is useful to realize that when $T_0 = 0$ that ω_y is an odd function of $x - 0.5d$ so that $\omega_y(x = 0.5d) = 0$ while v_z is an even function of $x - 0.5d$. Then Eq. (6) can be solved for spatial distributions of spin velocity by Runge–Kutta numerical integration from

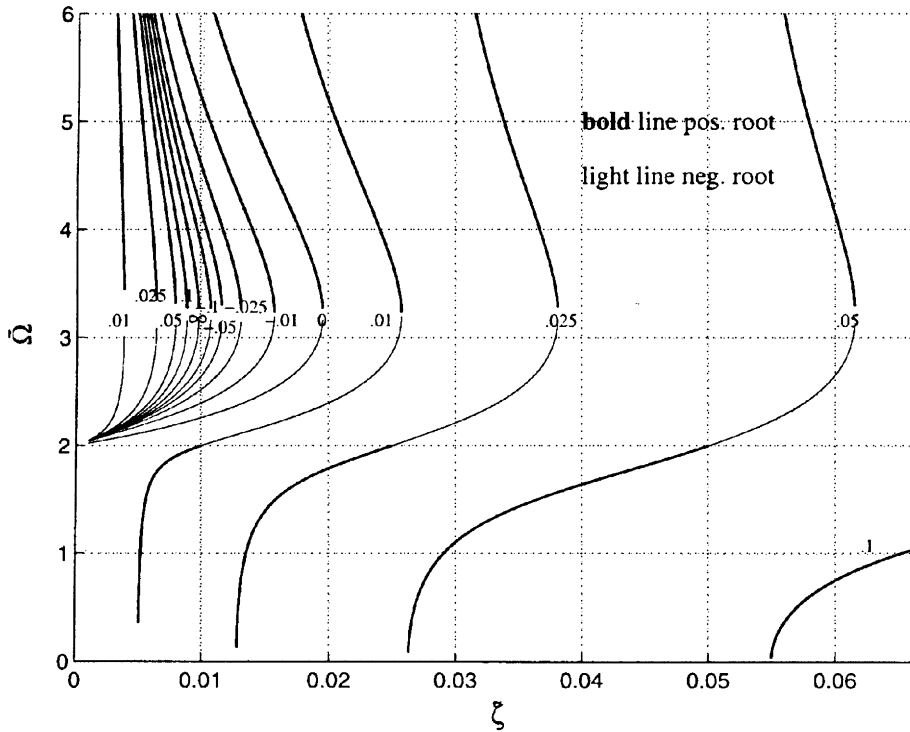


Fig. 1. Solutions for an axial magnetic field, $|\hat{B}_x| = 0$, $|\hat{H}_z| = H_0$, with $\chi_0 = 1$ and $\partial p/\partial z = \mu_0 H_0^2/d$ of non-dimensional frequency $\tilde{\Omega} = \Omega\tau$ as a function of non-dimensional vortex viscosity $\tilde{\zeta} = 2\zeta/(\mu_0 H_0^2 \tau)$ for various positive, zero, and negative values of $\tilde{\eta}_{eff} = 2\eta_{eff}/(\mu_0 H_0^2 \tau)$.

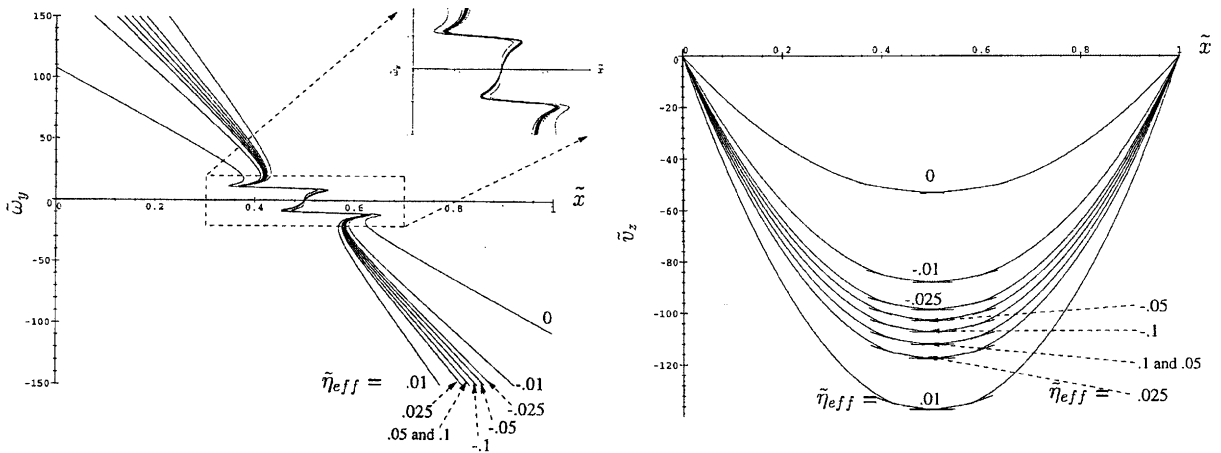


Fig. 2. Representative non-linear and multi-valued non-dimensional spatial distributions of spin velocity, $\tilde{\omega}_y = \omega_y \tau$, and linear velocity, $\tilde{v}_z = v_z \tau/d$, with $\Omega\tau = 10$ for various positive, zero, and negative non-dimensional values of $\tilde{\eta}_{eff} = 2\eta_{eff}/(\mu_0 H_0^2 \tau)$.

$\omega_y(x = 0.5d) = 0$ to yield non-linear and multi-valued solutions near $x = 0.5d$ as shown in Fig. 2. The flow velocity distribution, \tilde{v}_z , can then be numerically determined from Eq. (6) as a function of $\tilde{\omega}_y$, and is also parametrically plotted as a function of $\tilde{x} = x/d$ in Fig. 2. The essentially parabolic like profile is similar to that of usual Poiseuille pipe flow, but with magnetic field dependent effective viscosity. These multi-valued

solutions are probably non-physical due to the assumption that the spin viscosity is zero. However, they do point to the possibility of unusual flow behavior.

Acknowledgements

This work has been partially supported by a grant from the Exxon Education Foundation's Research and Training Program. Stimulating conversations with Dr. R.E. Rosensweig are greatly appreciated.

References

- [1] M. Zahn, P.N. Wainman, *J. Magn. Magn. Mater.* 122 (1993) 323.
- [2] G.H. Calugaru, C. Cotae, R. Badescu, V. Badescu, E. Luca, *Rev. Roum. Phys.* 21 (1976) 439.
- [3] R.E. Rosensweig, *Ferrohydrodynamics*, Cambridge University Press, Cambridge, 1985 (Chapter 8).
- [4] M.I. Shliomis, K.I. Morozov, *Phys. Fluids* 6 (8) (1994) 2855.
- [5] M. Zahn, D.R. Greer, *J. Magn. Magn. Mater.* 149 (1995) 165.
- [6] L.L. Pioch, MIT Masters of Engineering Thesis, June 1997.
- [7] J.-C. Bacri, R. Perzynski, M.I. Shliomis, G.I. Burde, *Phys. Rev. Lett.* 75 (11) (1995) 2128.